The speed of choice acclimation: implications for reference-dependent decision making*

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Abstract

In this paper, I compare two models of reference-dependent preferences by Kőszegi and Rabin (2007), which differ in whether individuals update their expectations after making an anticipated decision. This process is called choice acclimation. The hypothesis is that choice acclimation takes time, so that the choice-acclimated model applies in the long run and the unacclimated model in the short run. I apply both models to the decision of sports fans to attend a professional game. Using data on more than 50,000 European football games and 10,000 NBA basketball games, I find evidence that fans are loss averse and that they adapt to their decision even when the outcome is realised only three weeks after decision making.

1 Introduction

In the presence of risk, explanations of economic behaviour based on expected utility theory are often unsatisfactory. An important intuition that is not captured by the expected utility framework is that outcomes are frequently evaluated by comparison to a reference point. For example, individuals are generally found to be more sensitive to losses than to gains. Similarly, an expense may be viewed more negatively when it is unexpected. Issues such as these are addressed in the literature on prospect theory[1], starting with the work of Kahneman and Tversky (1979). More recently, Kőszegi and Rabin (2006, 2007) proposed a theory of reference-dependent preferences which has spawned numerous applications. In this theory, the reference point is equated with rational expectations of future outcomes. Individuals maximise a reference-dependent utility function given their recent expectations, resulting in what is dubbed a personal equilibrium.

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[1]For a recent review of prospect theory, see Barberis (2013).

In cases where a decision is anticipated, Kőszegi and Rabin (2007) distinguish between two equilibrium concepts: unacclimated personal equilibria (UPE), which are appropriate when an individual is unable to commit to a decision until slightly before the outcome is realised, and choice-acclimated personal equilibria (CPE), which are appropriate when the individual is able to commit far in advance. The main difference is that individuals take their expectations as a given in unacclimated models, because there is not enough time to adjust, but incorporate their decision into their expectations in acclimated models. It remains an open question how quickly people adjust their expectations after making a decision, a process I call *choice acclimation*. Kőszegi and Rabin (2007) note: "Although our [UPE] model is ambiguous as to the interpretation of 'shortly before,' insurance choices on short-term rentals such as cars or skis probably best correspond to such a case".

Existing applications in the literature typically have a natural time scale, which provides some indication. I mention just a few examples. At the short end of the spectrum, Crawford and Meng (2011) use UPE to study the same-day decision of New York City cab drivers to keep working in the afternoon after a certain income has been earned in the morning. At the other end of the spectrum, Herweg et al. (2010) use CPE to study employment contracts where remuneration occurs long after the agent has accepted the contract. Macera (2012) studies contracts of a shorter duration with UPE, noting that a month is a reasonable period for UPE to apply.

Experimental results are mixed, but point towards a shorter time scale. In a lab experiment of 1 hour, Matthey and Dwenger (2007) find evidence that initial aspirations keep affecting reference states throughout the experiment, suggesting that reference states adjust slowly over time. In contrast, Gill and Prowse (2012) find that reference states update almost instantaneously during an experiment of 90 minutes. The experiment of Song (2015), spanning 24 hours, also indicates a fast adjustment speed. However, as of yet there is no empirical guidance on the speed of choice acclimation over longer time scales. Clearly, it would be desirable to observe the behaviour of individuals facing the same decision at different times before the outcome is realised. The relative predictive power of the two models (UPE and CPE) over time would then be an indication of the appropriate time scales.

In this paper, I identify the decision of sports fans to attend a live game as a possible candidate for such an experiment. Assuming that fans derive utility from seeing their home team win, it follows that the decision to attend is risky and depends on the perceived chances of a home win. The reference-dependent utility function of Kőszegi and Rabin (2006) has been successfully applied in this context to regular season games of American football (Card and Dahl, 2011) and baseball (Coates et al., 2014). These papers employ the equivalent of a CPE model. By contrast to regular season games, where the fixtures are known far in advance, the opposing sides in knock-out tournament games are known only when the previous round has finished. Hence, fans that care who the opposing sides are have only limited time to decide whether to attend. This means that they are unable to commit to a decision until shortly before the outcome is realised, although the decision might very well be anticipated. I call the time between the moment when the opposing sides are known and the start of the game the *decision window*. The decision window varies from a single day to a week in the NBA play-offs and from one to several weeks or more in European football tournaments such as the UEFA Champions League. By applying the UPE and CPE models to both sports, it is possible to probe the speed of choice acclimation over a wide range of time scales.

My models assume that the demand for tickets is influenced by the probability of each outcome (win, draw, or loss). In similar vein, an influential hypothesis in sports economics states that demand is positively related to uncertainty of outcome (Rottenberg, 1956; Neale, 1964). This means that a game whose outcome is almost certain attracts fever attendants than a game where both teams have a fair shot. Despite the ubiquity of this hypothesis, empirical evidence is mixed (Szymanski, 2003). Coates et al. (2014) were the first to explain this type of behaviour by developing a consumer-choice model based on Card and Dahl (2011). They use the reference-dependent utility function of Kőszegi and Rabin (2006) in order to account for consumer expectations and prove that when the marginal utility of an unexpected win is higher than that of an unexpected loss, the behaviour predicted by the uncertainty of outcome hypothesis follows. Note that this contradicts the usual prediction of prospect theory that consumers are loss averse. However, using data on Major League Baseball games, Coates et al. (2014) find evidence against the uncertainty of outcome hypothesis and in support of a loss averse version of their reference-dependent model. Coates et al. (2014) make no explicit reference to choice acclimation, but their model is equivalent to a CPE model. I derive an analogous UPE version and extend both models to allow for the possibility of draws, which are rare in baseball but quite common in other sports. I then use data on more than 50,000 European football and 10,000 NBA basketball games, which include both regular season and knock-out tournament games, in order to test the two models.

I introduce the theory in section 2, starting with a technical review of the equilibrium concepts of Kőszegi and Rabin (2007), followed by a derivation of the game-attendance models and a number of testable hypotheses. Section 3 begins with a description of the data set. I explain my reduced form estimation in section 3.2. Finally in section 3.3, I investigate how the two models compare as the decision window is varied, finding that the CPE model is appropriate when the the decision window is $d \geq 1$ days long. Section 4 is the conclusion.

2 Theory

When offered a bet where the outcome is a random variable X with $F(x) = P(X \leq x)$, the expected utility framework predicts that an individual with initial wealth w compares the expected utility of taking the bet,

$$\mathrm{E}\left[u(w+X)\right] = \int u\left(w+x\right)\ \mathrm{d}F(x),$$

with the utility $u(w)$ of their current wealth. Prospect theory modifies this in the following way. First of all, it introduces the notion of a reference point r and assumes that individuals derive utility from gains and losses relative to r. It also predicts that consumers are more sensitive to losses than gains and that this sensitivity is a diminishing function of the magnitude of the gain/loss. Finally, it predicts an overweighting of unlikely outcomes.

A major obstacle to applications of prospect theory concerns the choice of the reference point. Kőszegi and Rabin (2006) argue that the reference point should be equated with rational expectations, which results in a closed model. Abstracting away from probability weighting, they capture the remaining notions of prospect theory in a value function

$$v(x|r) = u(x) + \mu\left(u(x) - u(r)\right),$$

where the utility function $u(\cdot)$ is now called 'consumption utility' and $\mu(\cdot)$ is a strictly increasing 'gain-loss utility' function that is concave for positive values and convex for negative values (representing diminishing sensitivity) and steeper for negative values than positive values (representing loss aversion). In the most general case, the outcome distribution F is compared to a reference distribution R, resulting in the reference-dependent utility function

$$U(F|R) = \int u(x)\ \mathrm{d}F(x) + \int\int \mu\left(u(x) - u(r)\right)\ \mathrm{d}F(x)\mathrm{d}R(r).$$

Throughout this paper, I employ the commonly used linear parametrisation $u(x) = x$ and $\mu(x) = \eta x$ for $x \geq 0$ and $\mu(x) = \eta\lambda x$ for $x < 0$. Here, $\lambda \geq 1$ represents the degree of loss aversion and $\eta > 0$ the overall weight of gain-loss utility.

2.1 Personal equilibria

In order to predict the behaviour of individuals facing a choice set D of possible outcome distributions, Kőszegi and Rabin (2006) introduce the notion of a 'personal equilibrium'. A personal equilibrium (PE) occurs when an individual's optimal decision, conditional on their expectations, coincides with their expectations. As mentioned in the introduction, Kőszegi and Rabin (2007) make the further distinction between unacclimated personal equilibria (UPE) and choice-acclimated personal equilibria (CPE). In choice-acclimated models, the assumption is that individuals update their expectations to reflect their decision. This leads to the following definition:

Definition 1 (CPE). *A choice $F \in D$ is a CPE if $U(F|F) \geq U(F'|F')$ for all $F' \in D$.*

In unacclimated models, the reference point is fixed, so the decision criterion is different:

Definition 2 (UPE). *A choice $F \in D$ is a UPE if $U(F|F) \geq U(F'|F)$ for all $F' \in D$.*

In other words, a person must be willing to follow through with their decision once the reference point has been fixed. Occasionally, a situation may arise in which there are multiple UPEs, which means that there are multiple self-fulfilling expectations. Since the decision is anticipated, it is possible beforehand to rank the UPEs by comparing the utility that the agent derives when he follows through with his plans. This leads Kőszegi and Rabin (2006, 2007) to define a preferred personal equilibrium[2] (PPE):

Definition 3 (PPE)**.** *A choice* $F \in D$ *is a PPE if it is a UPE and* $U(F|F) \geq U(F'|F')$ *for all UPEs* $F' \in D$*.*

In my model of game attendance, the agent chooses either to attend a live game (lottery F) or to stay home (lottery G). There occasionally arises a situation in which the only UPE is a mixed strategy H_p, where the agent commits to attending the game with probability p. This equilibrium is unstable in the sense that, once the reference point has been set, any shock to the relevant variables will make the agent prefer lottery F or G above the mixed lottery H_p. In this circumstance, I assign a probability of $1/2$ to the person attending or not. In the next section, I describe the game attendance models in greater detail.

2.2 Game attendance

Following Card and Dahl (2011) and Coates et al. (2014), I assume that the amount of utility that consumers derive from watching a sports game depends on the outcome. I let U^w denote the utility associated with attending a game where the home team wins. Similarly, U^d and U^l denote the utility associated with draws and losses. When consumers decide not to attend a game, they experience a baseline reservation utility of v. Individuals with low values of v are die-hard fans and will not want to miss any games, whereas individuals with high values of v only attend exceptionally exciting games[3]. Consumers may also experience outcome-dependent utility at home: perhaps by watching the game on TV or learning the outcome in the papers. Because of the expensive ticket price and the atmosphere in the stadium, it seems likely that consumers attending the game are more emotionally invested in the outcome than those that stay home, so that U^l, U^d, U^w can be interpreted as the additional utility derived from seeing the outcome live relative to learning the outcome by other means.

Besides these immediate streams of consumption utility, consumers also experience a sensation of regret or relief when the outcome is different from what they expected. To model consumer expectations, we require the prior

[2]An alternative is to assign a probability of $1/n$ to each equilibrium when n is the number of UPEs. For an example of this type of approach, see Soetevent and Kooreman (2007). However, the aim of this paper is to compare the viability of the UPE/PPE and CPE concepts, as defined by Kőszegi and Rabin (2007).

[3]Season pass holders may also be assumed to have low values of v, representing a perceived sunk cost.

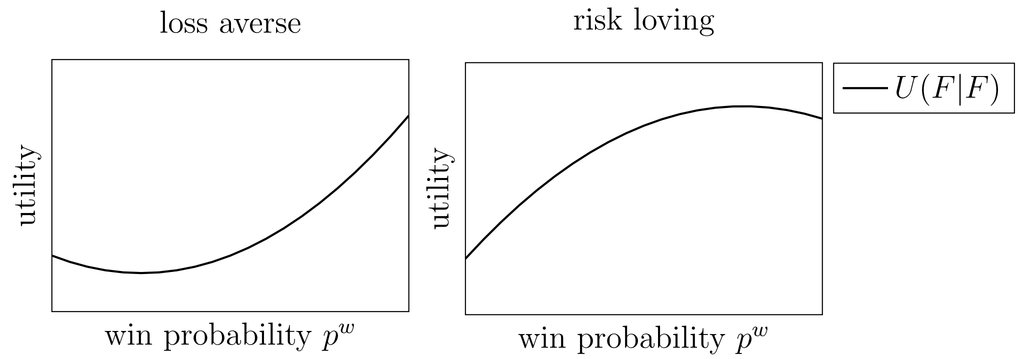

Figure 1: The reference-dependent utility of attending a sports game.

probabilities that consumers associate with a home win, a draw, and a loss. I denote these as p^w, p^d, p^l respectively. This is all the information needed to write down a reference-dependent utility function. When a consumer attends the game and expected to do so, his total utility is

$$U(F|F) = \sum_{i=w,d,l} p^i U^i + \sum_{i=w,d,l} \sum_{j=w,d,l} p^i p^j \ \mu \left(U^i - U^j \right).$$

Here, the first term is simply the expected consumption utility from attending the game, which is henceforth denoted by $\bar{U} := \sum_i p^i U^i$. The second term is gain-loss utility, which is accrued only when reality differs from expectation. I use the convention $i < j$ if and only if $U^i < U^j$, so that the piecewise linear function $\mu(\cdot)$ can be resolved into its linear components:

$$U(F|F) = \bar{U} + \eta(1 - \lambda) \sum_{i=w,d,l} \sum_{j<i} p^i p^j \left(U^i - U^j \right). \tag{1}$$

In figure 1, this quantity is plotted as a function of the win probability p^w in the two-outcome case ($p^d = 0$). The figure on the left shows that for a loss averse individual, the utility of attending the game given the expectation of attending reaches a minimum at an intermediate win probability. Fans prefer certain defeat over the likely possibility of being disappointed. Of course, a likely victory is preferred even more. The opposite happens for risk loving fans, as depicted on the right. They expect to receive the most pleasure from watching a game where the home team is likely but not certain to win, in accordance with the uncertainty of outcome hypothesis (Rottenberg, 1956).

The utility of a consumer that stayed home and expected to do so is simply

$$U(G|G) = v,$$

which is the reservation utility.

The CPE-framework of Kőszegi and Rabin (2007) applies when fans are able to commit to a decision, for example by buying a ticket, long before the game starts. It is assumed that fans adapt their expectations to reflect their decision, essentially forgetting that they could have stayed home if they made

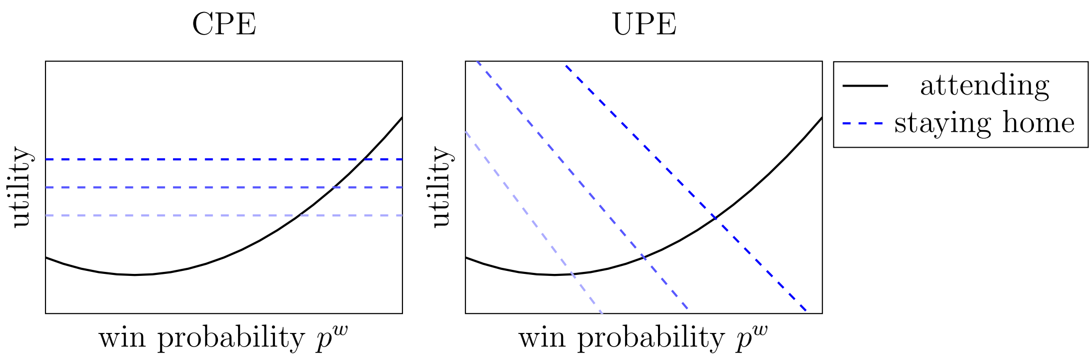

Figure 2: Attending the game is a PE whenever the utility of attending exceeds the utility of staying home, which is plotted for three different reservation utilities v.

plans to attend weeks ago and vice versa. The choice then simply depends on the following criterion:

$$U(F|F) \lesseqgtr U(G|G).$$

Attending the game is the unique CPE if and only if the reservation utility v is less than $U(F|F)$. Otherwise staying home is the CPE. Refer to the left panel in figure 2. Things become more complicated when fans are unable to commit until shortly before the game starts. In that case, even if someone expected beforehand that they would buy a ticket, they must still be willing to follow through once the fixture is announced. At that time, the expectation has been set and following through is preferable if and only if $U(F|F) \geq U(G|F)$, in which case attending the game is a UPE. See the right panel of figure 2. Because the expectation of attending has been set, the utility of staying home now depends on the win probability p^w.

This is not all, because staying home might also be a UPE. This occurs if and only if $U(G|G) \geq U(F|G)$. I already derived $U(F|F)$ and $U(G|G)$. The two remaining quantities are

$$\begin{aligned} U(G|F) &= v + \eta\lambda \sum_{U^i > v} p^i \left(v - U^i\right) + \eta \sum_{U^i < v} p^i \left(v - U^i\right), \\ U(F|G) &= \bar{U} + \eta \sum_{U^i > v} p^i \left(U^i - v\right) + \eta\lambda \sum_{U^i < v} p^i \left(U^i - v\right). \end{aligned}$$

In the UPE/PPE model, a fan will attend if and only if the decision to attend is the PPE, which is defined as the most preferable UPE. This can be expressed in terms of the reservation utility v. In Appendix A, I show that $U(F|F) \geq U(G|F)$, i.e. that attending the game is a UPE, if and only if $v \leq v^*$, where

$$v^* = \frac{\sum_i p^i w^i U^i}{\sum_i p^i w^i} + \eta(1-\lambda) \frac{\sum_i \sum_{j<i} p^i p^j \left(U^i - U^j\right)}{\sum_i p^i w^i}.$$

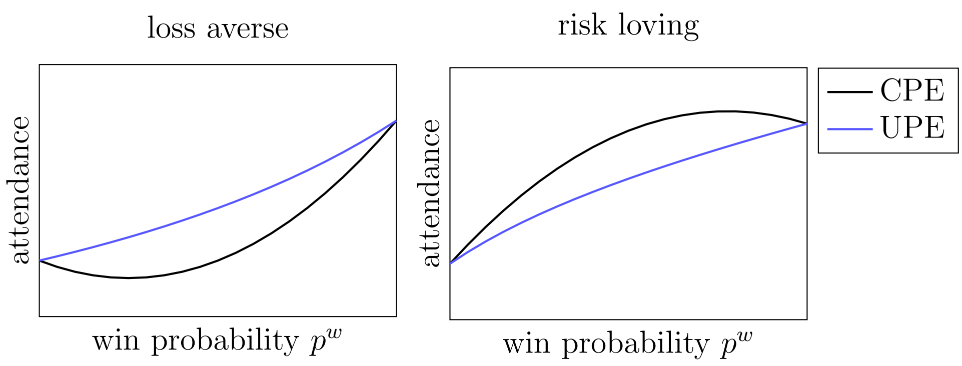

Figure 3: Game attendance according to the CPE and UPE models, assuming uniformly distributed reservation utilities. See section 2.3.

The first term is a weighted average utility with weights proportional to

$$w^i = 1 + \eta\lambda \qquad \text{for} \quad U^i > v^*,$$
$$w^i = 1 + \eta \qquad \text{otherwise},$$

so that positive outcomes ($U^i > v^*$) are weighted more heavily for loss averse individuals ($\lambda > 1$) and less for risk loving individuals. The second term is the gain-loss utility part of $U(F|F)$, attenuated by a factor $\sum p^i w^i \geq 1$. The implication is that reference-dependent effects are less important for the decision to follow through with attending (UPE criterion) than for the decision to attend in the committed case (CPE criterion). This results in a much flatter attendance curve in the UPE/PPE model, as seen in figure 3.

In Appendix A, I also show that $U(G|G) \geq U(F|G)$, or equivalently that staying home is UPE, if and only if $v \geq v^\dagger$, where

$$v^\dagger = \frac{\sum_i p^i \bar{w}^i U^i}{\sum_i p^i \bar{w}^i}, \qquad \text{with} \qquad \bar{w}^i = \begin{cases} 1 + \eta\lambda & \text{for} \quad U^i < v^\dagger, \\ 1 + \eta & \text{otherwise.} \end{cases}$$

For someone that stays home, a positive outcome is when the reservation utility exceeds the consumption utility they would have received from the game. We thus see that this is again a weighted average utility, with positive outcomes ($U^i < v^\dagger$) weighted more heavily for loss averse individuals ($\lambda > 1$) and less for risk loving individuals, attenuating any reference-dependent effects.

In summary, attending is a UPE if and only if $v \leq v^*$ and staying home is a UPE if and only if $v \geq v^\dagger$. These two inequalities completely determine whether someone would be willing to follow through with either decision, but which decision they ultimately make depends on the sign of $1 - \lambda$. First, consider the risk loving case $\lambda \leq 1$. I show in Appendix A that then $v^* \leq v^\dagger \leq U(F|F)$. It follows that attending the game is the unique UPE (and therefore PPE) for $v \leq v^*$ and staying home is the unique UPE (and PPE) for $v \geq v^\dagger$. In the interval $(v^*, v^\dagger)$, there is no pure strategy UPE. However, there exists a mixed strategy H_p in which consumers attend the game with

Table 1: Equilibria by level of reservation utility v

	v	**CPE**	**UPE(s)**	**PPE**
	Low	Attend	Attend	Attend
$\lambda \leq 1$	Intermediate	Attend	Unstable	Unstable
	High	Not attend	Not attend	Not attend
	Low	Attend	Attend	Attend
$\lambda \geq 1$	Intermediate	Not attend	Both	Not attend
	High	Not attend	Not attend	Not attend

Notes: Intermediate refers to values between v^* and $v^\dagger$. For the CPE column, low and high refer to values below and above $U(F|F)$, respectively. For the remaining columns, low and high refer to values below and above v^* and $v^\dagger$.

a certain probability p. The mixed strategy is not a distinct UPE if exactly one of the pure strategies is a UPE, but it is a distinct UPE if neither of the pure strategies is a UPE[4], which is true only in the interval $(v^*, v^\dagger)$. However, the mixed strategy equilibrium is unstable and any shock will make someone choose either decision with certainty. It is therefore more appropriate to assign probability 1/2 to both decisions for individuals with intermediate reservation utilities.

Next, consider the loss averse case $\lambda \geq 1$. It then follows that $U(F|F) \leq v^\dagger \leq v^*$. It is still true that attending the game is a UPE for $v \leq v^*$ and that staying home is a UPE for $v \geq v^\dagger$. Hence, in the intermediate interval $(v^\dagger, v^*)$, there exist two pure strategy equilibria[5]. As discussed in section 2.1, Kőszegi and Rabin (2007) predict that individuals will make the most preferable decision they are willing to follow through with. Hence, the secondary PPE constraint $U(F|F) \lesseqgtr U(G|G) = v$ comes into play. As it turns out, $U(F|F) \leq v^\dagger$, and so individuals with intermediate reservation utilities $v \in (v^\dagger, v^*)$ will always choose to stay home. Hence, fans attend if and only if $v \leq v^\dagger$. I summarise my findings in table 1.

2.3 Towards an empirical model

I now derive a reduced form model, which is used in the empirical analysis in section 3. Following Coates et al. (2014), I assume that the reservation utility v is i.i.d. uniform over the interval $[\underline{v}, \bar{v}]$ for a local population of N potential sports fans. In the CPE model, fans attend if and only if $v \leq U(F|F)$, so the expected attendance A is given by

$$\frac{A}{N} = \frac{U(F|F) - \underline{v}}{\bar{v} - \underline{v}}.$$

A similar expression follows for the UPE model with $U(F|F)$ replaced by $v^\dagger$ if there is loss aversion or $\frac{1}{2}v^\dagger + \frac{1}{2}v^*$ if there is not. Each possibility is plotted

[4] This is a general fact for any choice set with two pure strategies, as is proved in Appendix A.

[5] There is also a mixed strategy equilibrium, which is never prefered over a pure strategy alternative. See Appendix A.

in figure 3. The suppression of reference-dependent effects in the expressions for v^* and $v^\dagger$ is clearly visible.

An important unknown is the quantity

$$\delta = \frac{U^w - U^l}{\bar{v} - \underline{v}},$$

which represents the difference in attendance between games that are sure wins and sure losses. It may be the case that sure losses attract larger audiences, because people like to watch when a top club comes to town, even when the home team loses. However, holding the teams constant, it seems reasonable to expect fans to prefer the home team to win. It then follows from $U^i \in [\underline{v}, \bar{v}]$ that $0 < \delta < 1$.

I now derive linearised expressions for the expected attendance in terms of the outcome probabilities p^i for each model. For the CPE model, I use the approximation $\log(1 + x) \cong x$ to find

$$\begin{aligned} \log A &= \log N + \log\left(\frac{U(F|F) - \underline{v}}{\bar{v} - \underline{v}}\right) \\ &\cong \log N + \sum_{i=w,d,l} p^i \, \frac{U^i - \bar{v}}{\bar{v} - \underline{v}} + \eta(1 - \lambda) \sum_{i=w,d,l} \sum_{j<i} p^i p^j \, \frac{U^i - U^j}{\bar{v} - \underline{v}}. \end{aligned} \tag{2}$$

This expression includes a term for the total fan population, a term for consumption utility (linear in the probabilities), and a term for gain-loss utility (quadratic in the probabilities). The linear and quadratic dependence of log-attendance on the probabilities is key to the empirical work of Coates et al. (2014).

For the UPE model under loss aversion, I use the same approximation to work out the analogous expression:

$$\begin{aligned} \log A &= \log N + \log\left(\frac{v^\dagger - \underline{v}}{\bar{v} - \underline{v}}\right) \\ &\cong \log N + \sum_{i=w,d,l} p^i \bar{w}^i \, \frac{U^i - \bar{v}}{\bar{v} - \underline{v}} \\ &= \log N + \sum_{i=w,d,l} p^i \, \frac{U^i - \bar{v}}{\bar{v} - \underline{v}} + \sum_{i=w,d,l} p^i \eta \bar{\lambda}^i \, \frac{U^i - \bar{v}}{\bar{v} - \underline{v}}, \end{aligned} \tag{3}$$

where $\bar{\lambda}^i = \lambda$ if $U^i > v^\dagger$ and 1 otherwise. In this expression, we also see terms for total fan population, consumption utility, and gain-loss utility. However, because the binding constraint is $U(G|G) \leq U(F|G)$, consumers only compare each outcome of the game to the prospect of staying home, rather than the expected outcome of the game, so the quadratic dependence on probabilities

Table 2: Hypotheses for the reduced form model in section 3.2

Hypothesis	**Behaviour is best described by...**	**Predictions**
A	expected utility theory	$\beta_1 = \delta,\ B = 0$
B1	CPE with loss aversion.	$\beta_1 < \delta,\ B_{11} > 0$
B2	CPE without loss aversion.	$\beta_1 > \delta,\ B_{11} < 0$
C1	UPE with loss aversion.	$\beta_1 > \delta,\ B = 0$
C2	UPE without loss aversion.	$\beta_1 > \delta,\ B_{11} < 0$

Notes: The assumption that fans prefer a home win implies $0 < \delta < 1$.

is absent.

$$\begin{aligned}\log A &= \log N + \log\left(\frac{v^\dagger/2 + v^*/2 - \underline{v}}{\bar{v} - \underline{v}}\right)\\ &\cong \log N + \tfrac{1}{2}\log\left(\frac{v^\dagger - \underline{v}}{\bar{v} - \underline{v}}\right) + \tfrac{1}{2}\log\left(\frac{v^* - \underline{v}}{\bar{v} - \underline{v}}\right)\\ &\cong \log N + \left[1 + \frac{\eta(1+\lambda)}{2}\right]\sum_i p^i \frac{U^i - \bar{v}}{\bar{v} - \underline{v}} + \frac{\eta(1-\lambda)}{2}\sum_i \sum_{j<i} p^i p^j \, \frac{U^i - U^j}{\bar{v} - \underline{v}}.\end{aligned}$$

Note that there is a quadratic gain-loss utility term, resembling the simplified CPE model (2). There is also a linear gain-loss utility term, resembling the linearised UPE model under loss aversion (3). Since $\eta, \lambda \geq 0$, the linear terms are larger in magnitude than in the CPE model and the quadratic terms are smaller, but this may be difficult to detect.

Eliminating $p^l = 1 - p^w - p^d$ and setting $\mathbf{p} = (p^w, p^d)^{\mathrm{T}}$, I then model the log of attendance as a quadratic function of the probabilities:

$$\log A = \alpha + \boldsymbol{\beta} \cdot \mathbf{p} + \mathbf{p}^{\mathrm{T}} B\, \mathbf{p} + \boldsymbol{\theta} \cdot \mathbf{x} + \epsilon, \qquad \epsilon \sim \mathcal{N}\left(0, \sigma^2\right),$$

where B is a 2×2 symmetric matrix and $\mathbf{x}$ is a vector of control variables. Using the linearised expression above, one can write the coefficients α, $\boldsymbol{\beta}$, and B in terms of structural parameters[6]. One difficulty in making predictions about these coefficients concerns the relative magnitudes of the utilities U^l, U^d, U^w. The only assumption I make is that $U^l < U^w$, so that $0 < \delta < 1$. We can then distinguish the various models by looking at the coefficients β_1 and B_{11} of p^w and $(p^w)^2$, respectively. Refer to table 2 for an overview of all hypotheses. Notice that the UPE and CPE models without loss aversion (hypotheses B2 and C2) are indistinguishable. Furthermore, if $0 < \beta_1 < 1$, then it is impossible to say whether $\beta_1 \lesseqgtr \delta$.

3 Empirical analysis

In section 3.2, I describe a reduced form estimation based on the model introduced in the previous section. By and large, I am able to reproduce the

[6]Note that the terms $p^w p^l = p^w(1 - p^d - p^w)$ and $p^d p^l = p^d(1 - p^d - p^w)$ introduce terms proportional to $(p^w)^2$ and $(p^d)^2$, so that all coefficients of B are identified.

Table 3: Descriptive statistics by league

Sport	Country	League	N	Teams	Attendance
	England	Premier League	4 447	36	35 551
		Championship	5 819	54	17 537
		League One	3 362	56	7 817
		League Cup	512	85	18 621
	France	Ligue 1	4 104	37	20 434
		Ligue 2	2 795	41	7 874
		Coupe de la Ligue	231	47	9 922
	Germany	Bundesliga	3 385	34	42 626
		2. Bundesliga	3 201	49	16 674
		DFB-Pokal	86	28	44 018
Football	Italy	Serie A	4 532	39	23 359
		Serie B	3 189	59	5 986
		Coppa Italia	244	58	14 578
	Netherlands	Eredivisie	3 588	27	18 941
		Jupiler League	2 835	32	3 792
	Portugal	Primeira Liga	2 249	29	12 284
	Russia	Premier Liga	1 631	22	13 210
	Spain	La Liga	4 633	38	28 558
		Segunda División	3 763	56	8 252
		Copa del Rey	417	50	20 747
	International	Europa League	645	116	26 266
		Champions League	775	59	48 428
Basketball	U.S.	NBA	10 045	30	17 422

results of Coates et al. (2014), which pertain to regular season games of Major League Baseball, using my data on football and basketball games, suggesting that their model applies to a wider context of sports. In section 3.3, I address the issue of the speed of choice acclimation by testing whether the linear and quadratic coefficients in the reduced form model depend on the length of the decision window. First, I proceed with a description of the data set.

3.1 Data

Data on the number of attendants to 56 326 football games in 22 major European leagues were retrieved from `www.voetbal.com`. The data set covers 12 seasons between 2004 – 2016. Additionally, attendance records for 10 045 NBA basketball games played in the period 2009 – 2017 were obtained from `www.basketball-reference.com`. See table 3 for an overview of the leagues in the data set. A list of stadium capacities was compiled from public records. The outcome probabilities are estimated using betting odds, which were all retrieved from `www.oddsportal.com`. This website aggregates data from a large number of online bookmakers. See table 4 for further descriptive statistics by sport.

Betting odds are commonly used in the sports literature to estimate out-

Table 4: Descriptive statistics by sport

	Football		**Basketball**	
Variable	**Mean**	**St. Dev.**	**Mean**	**St. Dev.**
Attendance	19 284	16 269	17 422	2 477
Win probability	0.45	0.15	0.59	0.20
Draw probability	0.26	0.04	–	–
Loss probability	0.27	0.12	0.41	0.20
Home score	1.52	1.27	101.9	12.2
Away score	1.12	1.10	99.1	12.0
Observations	56 326		10 045	

come probabilities. A number of different methods are in use (Štrumbelj, 2014). The aim is to find the objective outcome probabilities p^i using the publicly posted betting odds o^i, where as always $i \in \{w, d, l\}$. I denote the inverse betting odds by $\pi^i = 1/o^i$ and the book sum by $\Pi = \sum_i \pi^i$. The most basic method, which is used by Coates et al. (2014), simply corrects for the bookmaker's take by normalisation:

$$p^i = \frac{\pi^i}{\Pi}. \tag{BN}$$

Call this method basic normalisation (BN). A more sophisticated method, based on a model by Shin (1991), assumes that a certain fraction z of the population has inside information and that bookmakers are profit-maximising insiders. The implied probabilities are

$$p^i = \frac{\sqrt{z^2 + 4(1-z)\pi^i/\Pi} - z}{2(1-z)}.$$

The proportion of insiders z can be computed from $1 = \sum_i p^i$ by fixed point iteration. For more details on this model, see Štrumbelj (2014). Finally, a third method (LR) makes use of an ordered logistic (or probit) regression of known historical outcomes on posted bookmaker odds (Forrest and Simmons, 2002; Forrest et al., 2005). Štrumbelj (2014) compares these three methods using a data set covering all major European football leagues, largely overlapping with my data. He advocates the use of SN, citing accuracy and robustness. I reproduce a part of his analysis using my own data. See table 5. Using a χ^2-test, I test the hypothesis that the derived probabilities are equal to the observed frequencies. This hypothesis can only be rejected for BN ($p = 0.0038$). Note that BN is more biased than SN by a factor of three and SN is more biased than LR by another factor of four. Moreover, all methods underestimate the home win probability. These findings are consistent with Štrumbelj (2014). As a robustness check, I used all three methods in my analysis, finding no major differences. In the following subsections I report the results obtained using SN.

A second issue with the relationship between betting odds and outcome probabilities concerns a different type of bias, reported by Forrest and Simmons (2002). They found that bookmakers posted less favourable bets for

Table 5: Biases for the three probability models

Method	Summed Absolute Bias	Mean bias by outcome		
		Home win	Draw	Away win
BN**	0.0198	−0.00991	0.00502	0.00489
SN	0.00734	−0.00380	0.00159	0.00195
LR	0.00167	−0.000836	0.000637	0.000198

Notes: BN: Basic normalisation, SN: Shin's model, LR: Logistic regression. The Summed Absolute Bias is the sum of the absolute values of the home win, draw, and away win biases.
** Significantly biased at the 1% level.

clubs with smaller numbers of supporters. I reproduced their analysis (not reported), finding no significant evidence for this type of bias. Hence, no further corrections were made.

3.2 Reduced form estimation

I now return to the reduced form model introduced in section 2.3, this time being more precise by accounting for the censored nature of the data. Let A_i^* be the number of fans that wish to attend game i. Based on the preceding theory, I expect that A_i^* is a quadratic function of the outcome probabilities $\mathbf{p}_i = \left(p_i^w, p_i^d\right)^{\mathrm{T}}$ and would like to estimate the equation

$$\log A_i^* = \alpha + \boldsymbol{\beta} \cdot \mathbf{p}_i + \mathbf{p}_i^{\mathrm{T}} B\, \mathbf{p}_i + \boldsymbol{\theta} \cdot \mathbf{x}_i + \epsilon_i, \qquad \epsilon_i \sim \mathcal{N}\left(0, \sigma^2\right), \tag{4}$$

where B is a 2×2 symmetric matrix and $\mathbf{x}_i$ is a vector of team, season, and month indicator variables. In practice however, we do not observe A_i^*, because the number of attendants is capped by the capacity C_i of the stadium. This can be remedied by using a right-censored Tobit model, which entails treating A_i^* as a latent variable, related to the observed number of attendants by $A_i = \min\{A_i^*, C_i\}$. Based on previous empirical work on outcome uncertainty (Coates et al., 2014; Czarnitzki and Stadtmann, 2002), one ought to be suspicious of the assumption of constant variance σ^2. Indeed, heteroskedasticity turns out to be a problem for the basic model (4). I looked for possible sources of misspecification using the conditional moment test proposed by Pagan and Vella (1989). See also Holden (2011) and Wells (2003) for an application. The primary advantage of this approach is that test statistics are readily obtainable through an auxiliary regression. I reject the null hypothesis of homoskedasticity ($p < 0.001$) and proceed with a conditional heteroskedastic Tobit model, where the scale factor of the error term is allowed to vary at the team-season level as a function of average attendance[7]. I also account for within-cluster correlation at the home team level by using cluster robust standard errors, obtained through block bootstrapping[8].

[7] See footnote **??** for the definition of average attendance.

[8] I use this approach rather than a robust estimator of the variance-covariance matrix, such as the Huber-White sandwich estimator, which does not address the fact that, for non-

In table 6, I report the results of three models using different sets of control variables. The first model uses only the logarithm of average attendance and a set of league indicators. This model is useful as a benchmark due to its simplicity. In Model 2, I replace average attendance with fixed effects for both home and away teams. Using team indicators as in Model 2 is preferable over using average attendance, on account of likelihood and information criteria (AIC and BIC). In Model 3, I include both average attendance and team, league, and month indicators, as well as additional control variables discussed below. For all models considered, I found significant evidence that $B_{11} > 0$. We can thus outright reject hypotheses A, B2, C1, and C2 (see table 2), leaving hypothesis B1 as the only corroborated hypothesis. Another robust finding at the 1% significance level is that $\beta_1 < 0$, which is also consistent only with hypothesis B1. These results suggest that the CPE model with reference-dependent preferences and loss aversion offers the best description of attendance behaviour in football.

Following Coates et al. (2014), I also include proxies for the quality of play in Model 3. I use the average number of goals scored and allowed (i.e. goals of the opponent) during home games for the home team and the average number of goals scored and allowed during away games for the away team. Perhaps unsurprisingly, the results indicate that fans prefer games with better playing teams. However, the quality of the home team is approximately twice as important as the quality of the away team. This is not entirely in line with Coates et al. (2014), who find that the offensive capabilities (measured by number of goals scored) of the away team are equally important as those of the home team. The results in table 6 show no difference in the importance between offensive and defensive qualities (measured by goals allowed). Instead of goals scored/allowed, I also tested a model with the current standings of the home and away teams as control variables. Using both sets of control variables (both standings and numbers of goals) makes some of the control variables insignificant, although the primary coefficients of interest are still significant. For this reason, I only include the model with goals scored/allowed in table 6. The issue of how these results hold up as the number of days between subsequent games is varied is taken up in the next section.

3.3 Test of choice acclimation

In the preceding sections, I assumed that the same model applies to everyone. In that case, the empirical results suggest that the CPE model with loss aversion is most appropriate. However, recall that the UPE model is supposed to apply in the short run and the CPE model in the long run. For simplicity, assume that the UPE model applies when a decision is made whose outcome is realised $t \leq T$ days in the future and that the CPE model applies otherwise. If people are to some extent myopic, meaning that they are less likely to consider attending a certain a game if there is an earlier game that still has

linear models, the coefficient estimates are inconsistent in the presence of heteroskedasicity. See Greene (2003, p. 692).

Table 6: Reduced form model results

Variable	Football Coefficient	Football z value	Basketball Coefficient	Basketball z value
Home win pr. (β_1)	-0.968^{***}	(-3.863)	-0.0961	(-1.239)
Home win pr. sqd. (B_{11})	1.144^{***}	(4.800)	0.166^{**}	(1.988)
Draw pr. (β_2)	-0.226	(-0.442)	–	–
Draw pr. sqd. (B_{22})	0.479	(0.661)	–	–
Win×draw prs. (B_{12})	0.998^{***}	(2.693)	–	–
Log avg. attendance	0.455^{***}	(16.467)	0.616^{***}	(6.614)
Home avg. scored	0.0312^{***}	(3.484)	0.00508^{***}	(5.063)
Home avg. allowed	-0.0263^{***}	(-2.732)	-0.00586^{***}	(-4.766)
Away avg. scored	0.0224^{***}	(4.361)	0.00806^{***}	(7.418)
Away avg. allowed	-0.0192^{***}	(-4.107)	-0.00560^{***}	(-7.388)
Month indicators	11		11	
Weekday indicators	6		6	
Team indicators	938		60	
League indicators	21		6	
Parameters	985		84	
Pseudo R^2	0.957		0.353	
Observations	56 326		10 045	

Notes: These are estimation results of three heteroskedastic right-censored Tobit models with log of attendance as the dependent variable. The scale factor of the error term is allowed to vary as a function of average attendance. Results were obtained using the `R`-package `crch` (Messner et al., 2015).

*** Significant at the 1% level.

** Significant at the 5% level.

* Significant at the 10% level.

to be played, then we should see the CPE model become more appropriate as the number of days d between subsequent games increases[9].

The implication is that the model parameters vary with d. For instance, the quadratic coefficient B_{11} of $(p^w)^2$ is zero according to the UPE model with loss aversion, but positive according to the CPE model with loss aversion. Thus, one expects that B_{11} is an increasing function of the number of days d between games for $d \leq T$. Similarly, looking at the hypotheses in table 2, one expects the linear coefficient β_1 of p^w to be a decreasing function of d.

[9]To see this, let $p_t = p_t(d)$ be the probability that someone decides whether to attend the game t days in advance. The fraction of people who have decided within the first T days is then

$$f_T(d) = 1 - (1 - p_1)(1 - p_2) \cdots (1 - p_T).$$

The myopia assumption is that $p_i(d) < p_i(d')$ when $d < i$ and $d' \geq i$, which implies that $f_T(d)$ is monotonic decreasing in d for $d \leq T$ and constant for $d > T$. Hence, the fraction of people to whom the UPE model applies is larger if there is more time between subsequent games.

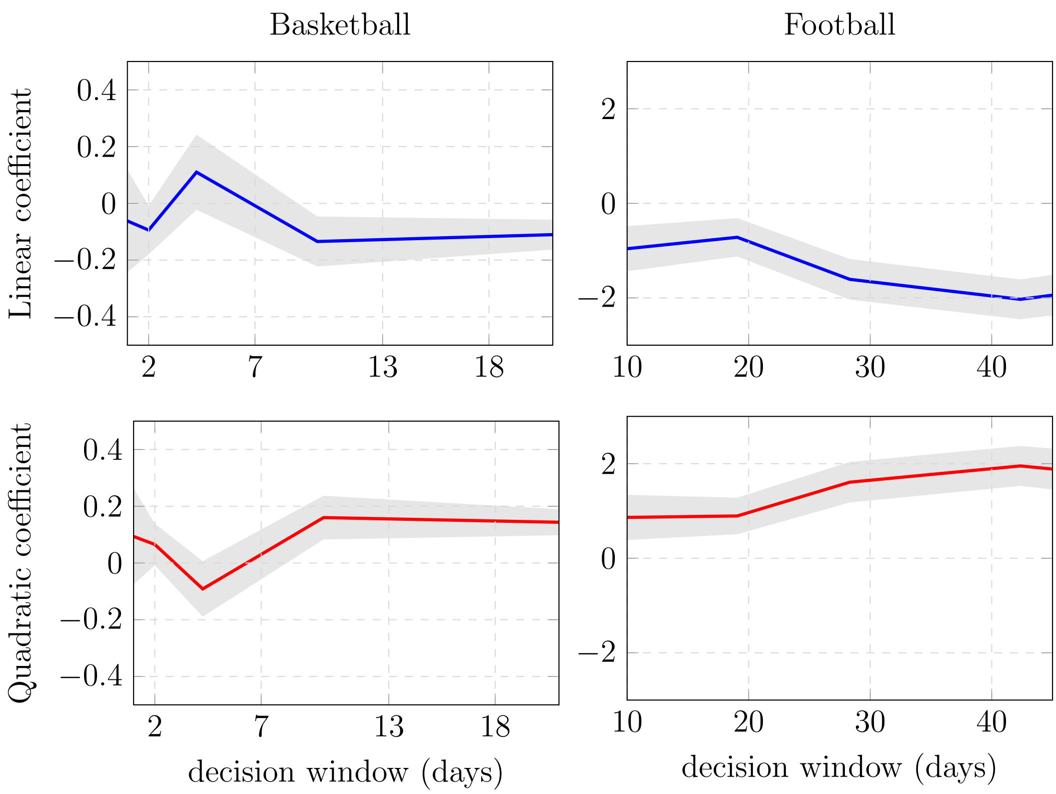

Figure 4: The linear coefficients (blue, top) and quadratic coefficients (red, bottom) of the win probability in the football and basketball models, as a function of the decision window.

To test these assertions, I divide the observations in different categories depending on the number of days between subsequent home games. I then evaluate the empirical models again, but this time include interaction terms of the coefficients of interest with indicators for the different categories. A complicating factor is that the number of observations differs wildly between the categories. Fortunately, the football and basketball data sets are complementary: most football games have $3 \leq d \leq 7$, whereas most basketball games have $d \leq 3$. The results are shown in table 7 and in figure 4.

In table 7, I have underlined the categories for which the data are only consistent with the CPE model with loss aversion at the 10% significance level. This is the case for almost all football categories with $3 \leq d \leq 10$ and the majority of basketball categories with $2 \leq d \leq 7$. Interestingly, the UPE model cannot be rejected for $d = 1$ and this holds for both the football and basketball models. For the football model, this might be explained by the limited number of observations ($N = 106$), but this is evidently not true for the basketball model ($N = 2\,991$). Looking at just the first week, we see that the linear coefficient β_1 decreases and the quadratic coefficient B_{11} increases with d. These results are in line with the expectation that the UPE model applies in the very short run and the CPE model in the long run. Roughly a quarter of the football games have values $d > 10$. Over these longer time scales, the results are mixed. See figure ??.

Table 7: Time dependence of the linear and quadratic coefficients β_1 and B_{11}

Football				**Basketball**			
Days	β_1	B_{11}	N	**Days**	β_1	B_{11}	N
≤ 14	-0.959^{*}	0.864	551	1	-0.062	0.093	79
$(14, 21]$	-0.718	0.894^{*}	557	2	-0.094	0.065	264
$(21, 33]$	-1.607^{***}	1.609^{***}	540	$(2, 6]$	0.110	-0.092	250
$(33, 54]$	-2.030^{***}	1.952^{***}	593	$(6, 19]$	-0.135	0.160^{*}	214
$(54, 67]$	-1.464^{**}	1.532^{**}	520	> 19	-0.091^{***}	0.130^{***}	11 338
> 67	-0.209^{***}	0.417^{***}	53 406				

Notes: These results were obtained by estimating interaction terms of the win probability and its square with the number of days between games.
*** Significant at the 1% level.
** Significant at the 5% level.
* Significant at the 10% level.

4 Conclusion

In this paper, I have developed a model of sports game attendance, using the reference-dependent decision framework of Kőszegi and Rabin (2006, 2007). My models are an extension of the baseball model of Coates et al. (2014). I was largely able to reproduce their results, using data on professional football games in Europe between 2004 – 2016 and NBA basketball games between 2009 – 2017, demonstrating that reference-dependent models of fan behaviour may be applicable to a wider context of sports. I used the models to derive estimates of the quantity $\eta(1-\lambda)$, representing the magnitude of effects due to loss aversion, finding significantly larger values for the football model than the basketball model.

Finally, I investigated how the appropriateness of the two models varies over time, finding that the UPE model may be adequate in the very short run, when the outcome is realised less than 1 day after the final decision is made, and that the CPE model is adequate for longer time scales. It seems that, in the context of the game attendance decision by sports fans, the process of choice acclimation occurs rather quickly. The approach used in this paper may be extended to different contexts where individuals make decisions at different times before the outcome is realised. For example, in the airline business, real time data is available on the price of refundable and non-refundable tickets. As the time of departure closes in, travellers make the risky decision between refundable and non-refundable tickets. In this situation, it is also possible to test the speed of choice acclimation if the outcome (the ticket is used or not) can be observed. Ultimately, without any evidence pointing to one of the two equilibrium frameworks, the best approach in practice may be to construct both a UPE and a CPE model and to see which of the two is a better fit to the data.

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A Proofs

Throughout this appendix, we shall use the notation

$$\lambda^i := \begin{cases} 1 & \text{if } v > U^i, \\ \lambda & \text{otherwise,} \end{cases} \qquad \bar{\lambda}^i := \begin{cases} \lambda & \text{if } v > U^i, \\ 1 & \text{otherwise,} \end{cases}$$

so that the utilities can be written as

$$\begin{aligned} U(F|F) &= \sum_{i=w,d,l} p^i U^i + \eta(1-\lambda) \sum_{i=w,d,l} \sum_{j<i} p^i p^j \left(U^i - U^j\right), \\ U(G|F) &= v + \sum_{i=w,d,l} p^i \eta \lambda^i \left(v - U^i\right), \\ U(F|G) &= \sum_{i=w,d,l} p^i U^i + \sum_{i=w,d,l} p^i \eta \bar{\lambda}^i \left(U^i - v\right), \\ U(G|G) &= v. \end{aligned}$$

The aim of this appendix is to prove a number of lemmas concerning the existence of UPEs. Throughout, we assume that $\eta, \lambda \geq 0$. We initially ignore the possibility of a mixed strategy, which is justified by Lemma 4.

Lemma 1. *Attending the game is a UPE if and only if $v \leq v^*$, where*

$$v^* = \frac{\sum_i p^i \left(1 + \eta\lambda^i\right) U^i}{1 + \sum_i p^i \eta \lambda^i} + \eta(1-\lambda) \frac{\sum_i \sum_{j<i} p^i p^j \left(U^i - U^j\right)}{1 + \sum_i p^i \eta \lambda^i}.$$

Proof. By definition, attending the game is a UPE if and only if $U(F|F) \geq U(G|F)$. Let $A(v) := U(F|F) - U(G|F)$. First of all, note that $U(G|F)$ is strictly increasing in v and $U(F|F)$ is independent of v. More precisely, $\mathrm{d}A(v)/\mathrm{d}v \leq -1$ for all v, so that $A(v)$ has a single zero for some $v^* \in \mathbb{R}$. We write $A(v)$ explicitly as a function of v:

$$\begin{aligned} A(v) &= U(F|F) - v - \sum_{i=w,d,l} p^i \eta \lambda^i \left(v - U^i\right) \\ &= U(F|F) - v \left[1 + \sum_{i=w,d,l} p^i \eta \lambda^i\right] + \sum_{i=w,d,l} p^i \eta \lambda^i U^i, \end{aligned}$$

such that $A(v) \geq 0$ if and only if $v \leq v^*$, with

$$\begin{aligned} v^* &= \frac{U(F|F) + \sum_i p^i \eta \lambda^i U^i}{1 + \sum_i p^i \eta \lambda^i} \\ &= \frac{\sum_i p^i \left(1 + \eta\lambda^i\right) U^i}{1 + \sum_i p^i \eta \lambda^i} + \eta(1-\lambda) \frac{\sum_i \sum_{j<i} p^i p^j \left(U^i - U^j\right)}{1 + \sum_i p^i \eta \lambda^i}. \end{aligned}$$

□

Lemma 2. *Staying home is a UPE if and only if $v \geq v^\dagger$, with $v^\dagger$ given by*

$$v^\dagger = \frac{\sum_i p^i \left(1 + \eta \bar{\lambda}^i\right) U^i}{1 + \sum_i p^i \eta \bar{\lambda}^i}. \tag{5}$$

Proof. By definition, staying home is a UPE if and only if $U(G|G) \geq U(F|G)$. Let $B(v) := U(F|G) - U(G|G)$, which we write as

$$B(v) = \sum_{i=w,d,l} p^i \left(U^i - v\right) + \sum_{i=w,d,l} p^i \eta \bar{\lambda}^i \left(U^i - v\right).$$

Observe that $B(v)$ is strictly decreasing in v. Furthermore, assume w.a.l.o.g. that $U^l \leq U^d \leq U^w$ (any other ordering will do). Then, it is easy to see that $B\left(U^w\right) \leq 0$ and $B\left(U^l\right) \geq 0$. Hence, $B(v)$ has a single zero for some $v^\dagger \in \left(U^l, U^w\right)$. We write the inequality $B(v) \leq 0$ as

$$\sum_{i=w,d,l} p^i \left(1 + \eta \bar{\lambda}^i\right) \left(U^i - v\right) \leq 0,$$

which is equivalent to $v \geq v^\dagger$, with

$$v^\dagger = \frac{\sum_i p^i \left(1 + \eta \bar{\lambda}^i\right) U^i}{1 + \sum_i p^i \eta \bar{\lambda}^i}.$$

□

Lemma 3. *If $\lambda \geq 1$, then $U(F|F) \leq v^\dagger \leq v^*$. If $0 \leq \lambda \leq 1$, then $v^* \leq v^\dagger \leq U(F|F)$.*

Proof. First, assume that $\lambda \geq 1$. To see that $U(F|F) \leq v^\dagger$, we use (1) and (5) to write the numerator of $v^\dagger - U(F|F)$ as

$$\begin{aligned}
&\sum_i p^i \left(1 + \eta \bar{\lambda}^i\right) \left[U^i - \sum_j p^j U^j - \eta(1-\lambda) \sum_x \sum_{y<x} p^x p^y \left(U^x - U^y\right)\right] \\
&\geq \sum_i p^i \left(1 + \eta \bar{\lambda}^i\right) \left[U^i - \sum_j p^j U^j\right] - \eta(1-\lambda) \sum_x \sum_{y<x} p^x p^y \left(U^x - U^y\right),
\end{aligned}$$

where we used the fact that $\sum_i p^i \left(1 + \eta \bar{\lambda}^i\right) \geq 1$ and $\eta(1-\lambda) \leq 0$. Continuing,

$$\begin{aligned}
&= \eta \sum_i \sum_{j \neq i} p^i p^j \bar{\lambda}^i \left(U^i - U^j\right) - \eta(1-\lambda) \sum_x \sum_{y<x} p^x p^y \left(U^x - U^y\right) \\
&= \eta \sum_i \sum_{j<i} p^i p^j \left(\bar{\lambda}^i - \bar{\lambda}^j\right) \left(U^i - U^j\right) - \eta(1-\lambda) \sum_x \sum_{y<x} p^x p^y \left(U^x - U^y\right) \\
&\geq \eta(1-\lambda) \sum_i \sum_{j<i} p^i p^j \left(U^i - U^j\right) - \eta(1-\lambda) \sum_x \sum_{y<x} p^x p^y \left(U^x - U^y\right) \\
&= 0,
\end{aligned}$$

where we used the fact that $\bar{\lambda}^i - \bar{\lambda}^j \geq 1 - \lambda$ for any i, j. With the requisite change of signs, the same argument can be used to show that $v^\dagger \leq U(F|F)$ if $0 \leq \lambda \leq 1$. This completes the first half of the proof.

For the second half, recall from Lemmas 1 and 2 that $A(v)$ and $B(v)$ are piecewise linear and strictly decreasing functions of v. Furthermore, $A(v^*) = B(v^\dagger) = 0$ by definition. Hence, if we can show that $B(v^*) \leq 0$, then it follows that $v^* \geq v^\dagger$ and vice versa. In general,

$$B(v) - A(v) = \sum_{i=w,d,l} p^i \eta \left(\bar{\lambda}^i - \lambda^i\right) \left(U^i - v\right) - \eta(1-\lambda) \sum_{i=w,d,l} \sum_{j<i} p^i p^j \left(U^i - U^j\right)$$

$$= \sum_{i=w,d,l} p^i \eta \left(1-\lambda\right) \left|U^i - v\right| - \eta(1-\lambda) \sum_{i=w,d,l} \sum_{j<i} p^i p^j \left(U^i - U^j\right).$$

Here we used the fact that that $\bar{\lambda}^i = \lambda$ if $v > U^i$ and 1 otherwise and similarly that $\lambda^i = \lambda$ if $v < U^i$ and 1 otherwise. One is easily convinced that $B(v) \geq A(v)$ for all v if $\lambda \geq 1$ and $B(v) \leq A(v)$ if $0 \leq \lambda \leq 1$. Indeed, because $B(v) - A(v)$ is a piecewise linear function of v which tends to $\text{sgn}(1-\lambda) \cdot \infty$ as $|v| \to \infty$, it suffices to consider the sign of the function at the points $v = U^l, v = U^d$, and $v = U^w$. A simple computation and collecting appropriate terms then yields the desired result. □

Lemma 4. *Let the choice set D consist of two pure strategies F and G and one mixed strategy H_p, where individuals choose F with probability p and G with probability $1-p$. Then, barring the degenerate case with $U(F|F) - U(G|F) + U(G|G) - U(G|F) = 0$, the following statements are true*

1. *For determining whether F and G are UPEs, one may ignore H_p,*
2. *H_p is not a distinct UPE if exactly one of the pure strategies is a UPE,*
3. *H_p is a distinct UPE if neither of the pure strategies is a UPE,*
4. *H_p is a distinct PPE if and only if neither of the pure strategies is a UPE.*

Proof. For the first part, assume that $U(F|F) \geq U(G|F)$. We will show that this implies $U(F|F) \geq U(H_p|F)$ for any $p \in [0,1]$. Hence, to show that F is a UPE, it suffices to prove that $U(F|F) \geq U(G|F)$. By symmetry, the analogous statement is true for G. We write

$$U(F|F) - U(H_p|F) = (1-p) \int u(x) \left[\mathrm{d}F(x) - \mathrm{d}G(x)\right]$$

$$+ (1-p) \int \mu\left(u(x) - u(r)\right) \left[\mathrm{d}F(x)\mathrm{d}F(r) - \mathrm{d}G(x)\mathrm{d}F(r)\right],$$

$$= (1-p)\left[U(F|F) - U(G|F)\right].$$

establishing the desired implication.

For the second part, recall that H_p is a UPE if and only if $U(H_p|H_p) \geq U(F|H_p)$ and $U(H_p|H_p) \geq U(G|H_p)$. Solving the equality $U(F|H_p) = U(G|H_p)$ for p yields

$$p = \frac{U(G|G) - U(F|G)}{U(F|F) - U(F|G) + U(G|G) - U(G|F)}, \tag{6}$$

except in the degenerate case. For this value of p, one finds that $U(H_p|H_p) = U(F|H_p) = U(G|H_p)$. It follows that if $0 < p < 1$ is a valid probability, then H_p is a distinct UPE. This is the only value of p for which both inequalities are satisfied, barring the degenerate case.

Assume that F is a UPE but G is not, so that by the first part $U(F|F) \geq U(G|F)$ and $U(G|G) < U(F|G)$. Using (6), we see that this implies that $p < 0$ or $p > 1$, so H_p is not a UPE for any $p \in (0, 1)$. The same happens if G is a UPE but F is not. Finally, assume that neither F nor G is a UPE. This means that $U(G|G) < U(F|G)$ and $U(F|F) < U(G|F)$, which implies that $0 < p < 1$, so H_p is a distinct UPE. This concludes the proof of the second and third parts.

For the last part, note that H_p is the only UPE if neither F nor G is a UPE. It is then also the PPE. If exactly one of the pure strategies is a UPE, then H_p is not even a UPE. Finally, if both F and G are UPEs, then we compute

$$\begin{aligned} U(F|F) - U(H_p|H_p) &= (1-p)\left[U(F|F) - U(F|G)\right], \\ U(G|G) - U(H_p|H_p) &= p\left[U(G|G) - U(G|F)\right]. \end{aligned}$$

Now suppose that H_p is the PPE for some $p \in (0, 1)$. Then we must both have $U(F|G) \geq U(F|F)$ and $U(G|F) \geq U(G|G)$. But that means that $U(F|F) - U(F|G) + U(G|G) - U(G|F) \leq 0$. But we also know that $U(G|G) \geq U(F|G)$, since G is a UPE. By (6), then $p \leq 0$, which is a contradiction. Hence, H_p is not a distinct PPE. □

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